MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

EEM2036 – ENGINEERING MATHEMATICS III

(All Sections/Groups)

18 OCTOBER 2016

2.30 PM – 4.30 PM

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of 7 **printed pages** (including cover page and formula sheets) with **four questions** only.

2. Attempt **ALL questions.** The distribution of marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

Solve the following system of differential equations:

$$\frac{dx_1}{dt} = x_1 + 4x_2$$
$$\frac{dx_2}{dt} = x_1 + x_2$$

Given the initial conditions $x_1(0) = 0$ and $x_2(0) = 2$. (20 marks)

Question 2

(a) Using the central difference formula, find the numerical derivative of $f(x) = x^2 + 5x - 10$ at the point $x = 2$ with $h = 0.1$ and $h = 0.05$. (5 marks)

(b) Given: $y' + \frac{x}{1+x^2} y = 3$; $y(0) = 1$, $h = 0.5$ where $0 \le x \le 1$.

(i) Solve by using Euler's method. (7 marks)

(ii) Hence, find the absolute error by comparing each of the solution found in part (i) with the exact value.

Hint: $\int \sqrt{1+x^2}\,dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left(x + \sqrt{1+x^2}\right) + C$ (10 marks)

(c) Consider the following system of linear equations:

$$x_1 + x_2 + 2x_3 = 1$$
$$2x_1 + x_2 = 2$$
$$x_1 + 2x_2 + 2x_3 = 3$$

Set up the Jacobi iterative equations and perform two iterations starting with the initial approximation (0,0,0). (8 marks)

Continued...

Question 3

(a) If V is a solid enclosed by a surface S oriented outward, evaluate

$$\iint_S \boldsymbol{F}(x,y,z)\cdot \boldsymbol{dS}$$

using Gauss' theorem, if $\boldsymbol{F} = 2x\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and the surface encloses a sphere of radius r. (10 marks)

(b) Given that $V(x,y,z) = 4xy^2 + xz - yx$ is a potential function of a vector field, $\boldsymbol{G}$. Prove that $\boldsymbol{G}$ is conservative. (10 marks)

(c) Evaluate the line integral $\oint_C 3ydx - 2xdy$ along C, using Green's theorem if C is a closed path consisting of the lines from (0,0) to (1,0) to (1,1) to (0,1) and (0,0). (10 marks)

Continued...

Question 4

(a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^{x^2+y^2}}{2} dydx$ by using polar coordinates. (5 marks)

(b) Find the volume of the solid that lies below the surface $z = 3 + \cos x + \cos y$ over the region R on the plane $z = 0$ bounded by the curves $x = 0, x = \pi, y = 0$ and $y = \pi$. (5 marks)

(c) Use the transformations $x + y = u$, $x - y = v$ and the Jacobian, $J = \frac{\partial(x,y)}{\partial(u,v)}$ to evaluate the following double integral over the square S shown in Figure 1.

$$\iint_R (x^2 + y^2)dxdy$$

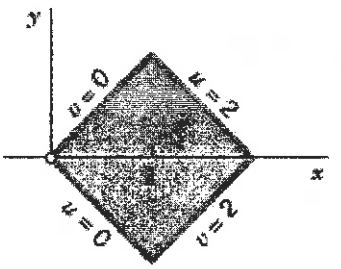

Figure 1 (10 marks)

Continued...

APPENDIX

TABLE OF FORMULAS

1. The *n*th Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1) \ldots (x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left| \frac{h^2}{6} f^{(3)}(c_x) \right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(a) + f(b)]$$

for some ξ in (a, b) and $h = b - a$, with the error term is $\left| \frac{h^3 f''(\xi)}{12} \right|$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term is $\left| \frac{(b-a)h^2 f''(\xi)}{12} \right|$.

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{2}$, with the error term $\left| \frac{h^5}{90} f^{(4)}(\xi) \right|$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term $\left| \frac{(b-a)h^4}{180} f^{(4)}(\xi) \right|$.

Continued ...

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2,...$$

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1),$$
$$k_3 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of Paper